This article develops a two-country monetary economy model in order to analyze the international monetary policy game between governments and the domestic monetary policy game between each government and its private sector. We prove that if governments can commit to their own private sectors, the cooperative equilibrium of the game between governments is for them to follow the Friedman rule. When governments lack such ability to commit, we find that the Friedman rule is more likely to be sustained in our open-economy model than in the closed-economy model of Ireland. (JEL E31, E52, E61)

I. INTRODUCTION

One recurring theme in the analysis of policy-making is that in many situations, policymakers can benefit from the opportunity to make a binding commitment to restrict their activities. One such situation is what is known as the “time consistency problem” in monetary policy-making. A government, or its monetary authority, out of good intention, may find it beneficial to generate unexpected inflation to fix some nonmonetary distortion in the economy, for example, goods market inefficiency due to monopoly power that is characterized by higher-than-efficient product prices. This first-best monetary policy, however, is not “time consistent” in the sense that firms foresee the government’s incentive to impose monetary surprises and set their nominal prices at a higher level. As a result, the government’s attempts to remedy the inefficiency in the goods market using monetary policy lead only to higher inflation, with the goods market inefficiency unchanged. Therefore, it is better for the government to make a commitment not to adopt any active monetary policy to fix the nonmonetary distortion, so that the second-best outcome with low inflation can be achieved.

For such a commitment by the government to work, it has to be credible. There does exist an implicit reputation mechanism that may make a monetary authority’s commitment credible. When a monetary authority repeatedly makes policies for many number of periods, a good reputation of sticking to its commitment is important for its long-run success. Even though a deviation from the announced second-best plan may benefit the monetary authority in the short run by achieving a temporary first-best result, it would destroy the authority’s reputation, and as a result, the economy would end up at an equilibrium with a high inflation that is dominated by the second-best result with low inflation. If the short-run benefits from deviation are exceeded by the long-run costs from loss of reputation, a rational government would honor its commitment and the private sector sees this. As a result, the second-best outcome is sustainable. Otherwise, the commitment of
not using surprise inflation is not credible, and the second-best outcome is not sustainable.

This article studies the "commitment problem" and the sustainability of the second-best outcome in an open-economy context. The main message of this article is that trade linkage between economies may be a solution to the commitment problem facing domestic monetary authorities. In essence, free trade facilitates a competitive mechanism among sovereign governments, which reduces the short-run benefits from surprise inflation and therefore makes the low-inflation policy more credible. Importantly, if sovereign governments cooperate with one another rather than compete, the benefit from international trade in terms of making governments' commitments more credible does not exist. Therefore, another lesson of the article is that cooperation among sovereign governments in the area of monetary policy-making may not be so desirable.

We build an open-economy version of Ireland's (1997) closed-economy environment. We add a second equally sized economy and allow trade between these economies. Ireland (1997) analyzed the closed economy, concentrating on issues of government commitment to households and the possibility of a reputational equilibrium in which the optimal policy under commitment can be sustained. Our open-economy extension allows us to look at two simultaneous monetary games: one between the two governments and the other between each government and its own private sector. Thus, this work follows in the line of previous work by Canzoneri and Henderson (1991), Henderson and Zhu (1990), and Kimbrough (1993). However, the underlying economics of our model varies significantly from that used by these authors in that our model is specified in terms of tastes and technologies. This microfounded approach has two advantages. First, alternative monetary policies can be evaluated in terms of their effects on welfare, measured using a representative household's utility function; there is no need to specify an ad hoc loss function for the monetary authority. Second, in a model with microfoundations, the parameters are those describing tastes and technologies; since these parameters can be reasonably calibrated based on findings from empirical studies, it is much easier to get a feel for whether the required conditions for reputational mechanisms are likely to hold.

To preview our results, we find that when two governments can commit to their respective private sectors, the cooperative equilibrium is that both governments contract money supplies so that the net nominal interest rate in both countries is 0, as required by the Friedman rule. Depending on the values of the parameters involved, the cooperative equilibrium may also be a Nash equilibrium. If that is the case, then there would be no need for any international agreements to facilitate cooperation among honest monetary authorities.

More significantly, we study the sustainability of the Friedman rule in the more realistic case where the governments lack a commitment technology. We find that the Friedman rule, the optimal policy when both governments can commit to their private sectors and at the same time can cooperate with each other, is more likely to be sustained in the open-economy model than in the Ireland's closed-economy model, if the governments do not (cannot) cooperate in generating inflationary surprises.

Underlying our main result is the idea of counterproductive cooperation between governments. The basic idea of counterproductive cooperation between monetary authorities has been made before (see Kehoe 1989; Rogoff 1985). What is new in this article is that the counterproductive cooperation takes a different notion and is developed in a model with firmer microfoundations. In this article, the optimal policy becomes less sustainable (in a repeated game) due to the cooperation between governments, while in the earlier studies on counterproductive cooperation, it is the time-consistent suboptimal policy (in a one-shot game) that becomes worse due to cooperation. On the other hand, as we said before, the parameters in a model with microfoundations are those describing tastes and technologies, and they can be reasonably calibrated based on findings in the empirical literature.

This article is organized as follows. The model is presented in the next section. In Section III, we show that when the governments can commit to their private sectors, the cooperative equilibrium of the game between the two governments is characterized by the Friedman rule. We also study the Nash equilibria of this game. In Section IV, we study the sustainability of the Friedman rule when
governments lack such commitment technology and compare it with the similar condition obtained within the closed-economy setting.

II. THE MODEL

The behavioral relations describe a two-country version of Ireland’s (1997) closed-economy setup.1 We use this model to investigate monetary policy interactions simultaneously between two sovereign governments and between each government and its own private sector. The model has utility-maximizing individuals, monopolistically competitive firms, and sticky output prices. Each government looks to maximize the utility function of the domestic representative agent, so that within a country, government and private objectives coincide.

We assume that there are two countries of equal size, a home country and a foreign country. Variables with tildes denote foreign variables. The behavioral relations in both countries are symmetric.2 Each country consists of three players: a government, a continuum of monopolistically competitive firms indexed by \( i \in [0, 1] \) for the home country and \( i \in [0, 1] \) for the foreign country, and a representative individual. The timing of events and roles of each player in this economy are described below.

Each government controls her money supply and makes a lump-sum transfer to the respective representative individual at the beginning of each date \( t = 0, 1, 2, \ldots \). This transfer is \((s_t - 1)M^s_t\) for the foreign government and \((\tilde{s}_t - 1)\tilde{M}^s_t\) for the foreign government, where \(M^s_t\) and \(\tilde{M}^s_t\) are, respectively, the per capita home money stock and the foreign money stock at the beginning of time \( t \) and \( s_t \) and \( \tilde{s}_t \) are, respectively, the gross money growth rates in the home and the foreign countries. So \( M^s_{t+1} = s_tM^s_t \) and \( \tilde{M}^s_{t+1} = \tilde{s}_t\tilde{M}^s_t \).

Firms are monopolistically competitive. They set output prices at the beginning of each period and are required to supply output at that set price for the entire period. So the home firm \( i \) and the foreign firm \( \tilde{i} \) enter period \( t \) with fixed nominal output prices \( P(i) \) and \( \tilde{P}(\tilde{i}) \), respectively. Firms in each country produce distinct perishable consumption goods; hence, goods are also indexed by \( i \) and \( \tilde{i} \). Monopolistic competition results in equilibrium output falling below the efficient level, while sticky prices allow unanticipated money to generate real effects on output.

The representative home individual enters period \( t \) with home money \( M_t \) and home (private) bonds \( B_t \). Similarly, the representative foreign individual enters period \( t \) with foreign money \( M_t \) and foreign (private) bonds \( B_t \). In equilibrium, money demand equals money supply, and private bonds are available in zero net supply. That is to say \( M_t = M_t^s, \tilde{M}_t = \tilde{M}_t^s \), and \( B_t = \tilde{B}_t = 0 \) for all \( t = 0, 1, 2, \ldots \). However, we can impose these equilibrium conditions only after the individual optimization problems are solved. The representative individual of each country is allowed to hold only domestic money and bonds. This assumption simplifies the analysis, but it also seems necessary for an analytical solution. It turns out that a crucial feature in constructing the monetary game is to relate the policy variables to the equilibrium values of the welfare variables that appear in the utility functions. This assumption makes such a connection possible.

The gross nominal interest rates are \( R_t \) for the home bonds and \( \tilde{R}_t \) for the foreign bonds. A home (or foreign) bond with a face value of \( B_{t+1} \) (or \( \tilde{B}_{t+1} \)) paying a gross nominal interest rate \( R_t \) (or \( \tilde{R}_t \)) at time \( t + 1 \) will cost the representative (home or foreign) individual \( B_{t+1} + R_t B_{t+1} \) (or \( \tilde{B}_{t+1} + \tilde{R}_t \tilde{B}_{t+1} \)) in home (or foreign) currency at time \( t \). The representative individual in each country also consumes both home goods (goods produced by home firms) and foreign goods (goods produced by foreign firms). However, purchasing nondomestic goods first involves an exchange of domestic money for foreign money.

The timing of transactions is as follows. At the beginning of each time \( t \), the representative home individual receives a nominal transfer \((s_t - 1)M^s_t\) from the home government; likewise, the representative foreign individual receives a nominal transfer \((\tilde{s}_t - 1)\tilde{M}^s_t\) from his government. Then, bonds \( B_t \) and \( \tilde{B}_t \) mature, so that the money holdings are \( M_t + (s_t - 1)M^s_t + B_t \) in home currency for the home individual and \(\tilde{M}_t + (\tilde{s}_t - 1)\tilde{M}_t^s + \tilde{B}_t \).
in foreign currency for the foreign individual, where \( \bar{M}_t \) and \( M_t \) are, respectively, the money holdings of the home individual and of the foreign individual left from the transactions at time \( t - 1 \). The representative home and foreign individuals then use these money holdings to purchase new bonds and consumption goods. In doing so, they face their respective cash-in-advance (CIA) constraints:

**Home CIA:**  
\[
M_t + (x_t - 1)M_t^s + B_t - B_{t+1}/R_t \geq \int_0^1 P_t(i) c^h(i) di + e_t \int P_t(i) \bar{c}^h(i) d\bar{i},
\]

and

**Foreign CIA:**  
\[
\bar{M}_t + (\bar{x}_t - 1)\bar{M}_t^s + \bar{B}_t - \bar{B}_{t+1}/\bar{R}_t \geq \int \bar{P}_t(i) \bar{c}^h(i) d\bar{i} + 1/e_t \int P_t(i) c^h(i) di.
\]

Here, \( e_t \) is the nominal exchange rate, which is defined as the home currency price of one unit of foreign currency, and \( c^h(i) \) and \( \bar{c}^h(i) \) denote the representative home individual’s consumption of goods \( i \) and \( \bar{i} \), with fixed prices \( P_t(i) \) and \( \bar{P}_t(i) \), respectively. Similarly, \( c^l(i) \) and \( \bar{c}^l(\bar{i}) \) are representative foreign individual’s consumption of goods \( i \) and \( \bar{i} \).

Next, the representative home individual supplies \( n_t(i) \) units of labor to domestic firm \( i \in [0,1] \) and receives a nominal wage rate \( W_t \), and the representative foreign individual supplies \( \bar{n}_t(\bar{i}) \) units of labor to foreign firm \( \bar{i} \in [0,1] \) and receives nominal wage rate \( \bar{W}_t \). It is assumed that there is no employment overseas. Hence, per capita labor supplies in both countries are, respectively,

\[
n_t = \int_0^1 n_t(i) di,
\]

\[
\bar{n}_t = \int_0^1 \bar{n}_t(\bar{i}) d\bar{i},
\]

and the wage payments received by representative home and foreign individuals are \( n_t W_t \) and \( \bar{n}_t \bar{W}_t \), respectively.

We assume that only home individuals may own home firms and only foreign individuals may own foreign firms. Firms pay out all profits as dividends to the representative individuals. We also assume that firms in each country produce outputs with linear technology that yields one unit of output for every unit of labor input. For the home firms, therefore, dividends paid out are

\[
D_t(i) = [P_t(i) - W_t]y^D_t(i, P_t(i)), \quad i \in [0,1].
\]

Similarly, foreign firms pay out dividends according to

\[
\bar{D}_t(\bar{i}) = [\bar{P}_t(\bar{i}) - \bar{W}_t]y^D_t(\bar{i}, \bar{P}_t(\bar{i})), \quad \bar{i} \in [0,1].
\]

Firms \( i \) and \( \bar{i} \) face downward-sloping output demand curves \( y^D_t(i, P_t(i)) \) and \( \bar{y}^D_t(\bar{i}, \bar{P}_t(\bar{i})) \), respectively. These arise due to monopolistic competition. The curves \( y^D_t(i, P_t(i)) \) and \( \bar{y}^D_t(\bar{i}, \bar{P}_t(\bar{i})) \) are determined by representative individuals’ demands for goods \( i \) and \( \bar{i} \) at prices \( P_t(i) \) and \( \bar{P}_t(\bar{i}) \), respectively.

At the end of period \( t \), the representative home and foreign individuals have unspent cash, wage receipts, and dividend as sources of money holdings to be carried into period \( t + 1 \). These money holdings are \( M_{t+1} \) for the home individual and \( \bar{M}_{t+1} \) for the foreign individual. The budget constraint (BC) for home individual is

**Home BC:**  
\[
M_t + (x_t - 1)M_t^s + B_t - B_{t+1}/R_t + n_t W_t + \int D_t(i) di \geq \int P_t(i) c^h(i) di + e_t \int \bar{P}_t(\bar{i}) \bar{c}^h(\bar{i}) d\bar{i} + M_{t+1},
\]

and it is exactly symmetric for the foreign BC.

Households receive utility from both domestic and foreign goods. The preferences

3. To focus on a firm’s pricing decision, we have suppressed the other arguments in the demand function for the firm’s product. So only its own price is explicitly left as an argument.
of representative home and foreign individuals are denoted by the utility functions

\begin{equation}
\sum_{i=0}^{\infty} \beta^i [K(c_i^t)^\alpha (c_i^t)^\delta - n_t],
\end{equation}

\begin{equation}
\sum_{i=0}^{\infty} \beta^i [K(z_i^t)^\alpha (z_i^t)^\delta - \bar{n}_t],
\end{equation}

respectively, where $0 < \beta < 1$, $\alpha > 0$, $\delta > 0$, $\alpha + \delta < 1$, and $K > 0$. The composite goods $c_i^h$, $z_i^h$, $c_i^f$, and $z_i^f$ are defined by

\begin{align*}
c_i^h &= \left[ \int_0^1 c_i^h(i)^{(0-1)/\alpha} di \right]^{0/(0-1)}, \\
z_i^h &= \left[ \int_0^1 z_i^h(i)^{(0-1)/\alpha} di \right]^{0/(0-1)}, \\
c_i^f &= \left[ \int_0^1 c_i^f(i)^{(0-1)/\alpha} di \right]^{0/(0-1)}, \\
z_i^f &= \left[ \int_0^1 z_i^f(i)^{(0-1)/\alpha} di \right]^{0/(0-1)},
\end{align*}

where $0 > 1$. These specific functional forms imply a constant elasticity of substitution (CES) between goods produced in a country. If $0$ is large, then these goods are close substitutes. In the utility functions, Equations (8) and (9), domestic goods are allowed to have a different weight than nondomestic goods (when $\alpha \neq \delta$). At the same time, these specifications assume a symmetry between the two countries, which is consistent with our modeling assumption of two equal-sized and basically equivalent nations.

Before proceeding with our analysis, we define the following scaled nominal variables:

\begin{align*}
m_t &= M_t / M^s_t, \\
b_t &= B_t / M^s_t, \\
p_t(i) &= P_t(i) / M^s_t, \\
w_t &= W_t / M^s_t, \\
d_t(i) &= D_t(i) / M^s_t, \\
\bar{m}_t &= \bar{M}_t / M^s_t, \\
\bar{b}_t &= \bar{B}_t / M^s_t, \\
\bar{p}_t(i) &= \bar{P}_t(i) / M^s_t, \\
\bar{w}_t &= \bar{W}_t / M^s_t, \\
\bar{d}_t(i) &= \bar{D}_t(i) / M^s_t.
\end{align*}

Note that $m_t$ is the ratio of representative home individual’s money holdings to the per capita money stock of the home country at time $t$; hence, it can be normalized to one in equilibrium in which all domestic individuals hold an equal amount of money. Similarly, $\bar{m}_t$ can be normalized to one in equilibrium. In addition to the above definitions of the scaled nominal variables, let $k_t = M^s_t / M^s_t$, the ratio of per capita money stock of foreign country to home country.

Now we are able to rewrite the representative home and foreign individuals’ CIA and BCs in terms of the scaled variables. For example, home CIA and BC can be written as:

Home CIA: 

\begin{align}
m_t + (x_t - 1) + b_t - x_t b_t+1 / R_t \\
&\geq \int_0^1 p_t(i) c_i^h(i) di + k_t e_i \int_0^1 \bar{p}_t(i) ^f c_i^f(i) di
\end{align}

and

Home BC:

\begin{align}
m_t + (x_t - 1) + b_t - x_t b_t+1 / R_t \\
&+ n_t w_t + \int_0^1 d_t(i) di \geq \int_0^1 p_t(i) c_i^h(i) di \\
&+ k_t e_i \int_0^1 \bar{p}_t(i) ^f c_i^f(i) di + x_t m_t+1.
\end{align}

The dividend payments of the representative home firm $i$ can also be expressed as

\begin{equation}
d_t(i) = [p_t(i) - w_i] y_i^d(i, p_t(i)), \quad i \in [0, 1],
\end{equation}

where $y_i^d(i, p_t(i))$ is the representative individuals’ demands for goods $i$ at scaled prices $p_t(i)$.

III. NASH AND COOPERATIVE EQUILIBRIA WHEN GOVERNMENTS ARE CREDIBLE

If we disregard any problem with time consistency of optimal policies, it seems reasonable to expect that the cooperative equilibrium is for both governments to follow the Friedman rule, that is, each government contracts money supply so that the net nominal
interest rate is 0. We derive this result later, but the basic reason is that monetary policy cannot alleviate the monopoly distortion implied by imperfect competition in the goods market. Thus, the most that the two governments can do with monetary policy is to make sure that the real economy is not distorted by monetary policy, that is, to implement the Friedman rule, just as in the case of the closed economy. A new dimension added by the open-economy extension is that one government’s policy may have externalities on another through its impact on the exchange rate. Consequently, the Nash and the cooperative equilibria tend to differ from each other. Indeed, in previous studies on this topic that employ only aggregate relationships, the Nash equilibrium is usually characterized by a higher inflation rate. Therefore, these earlier studies have generally emphasized the benefits of cooperation among sovereign governments. Using a model grounded in microfoundations, we find that there exist situations in which the Nash and the cooperative equilibria are identical.

We begin with the optimization problems of private agents and find the representative individuals’ and firms’ best actions. We assume that both governments can credibly commit to domestic private agents by setting a sequence of gross money growth rates \( x = \{x_t, t = 0, 1, 2, \ldots \} \) of the home government and \( \bar{x} = \{\bar{x}_t, t = 0, 1, 2, \ldots \} \) of the foreign government, where \( x_t \in [\beta, \bar{x}] \) and \( \bar{x}_t \in [\beta, \bar{x}] \) for all \( t \). These assumptions mean that individuals take as given the entire paths of \( x \) and \( \bar{x} \) when solving their maximization problems. The bounds on gross money growth rates ensure the existence of a monetary equilibrium. As shown below, the lower bound, \( \beta \), on both \( x_t \) and \( \bar{x}_t \), helps guarantee that the net nominal interest rates \( R_t - 1 \) and \( \bar{R}_t - 1 \) are nonnegative in equilibrium. As for the upper bound, \( \bar{x} < \infty \), this is introduced following Calvo (1978) in order to guarantee that private agents never abandon the use of money altogether.

The representative home individual’s problem is to maximize his lifetime utility given in Equation (8), subject to the definitions of composite goods \( c_t^h \) and \( c_t^b \) (the first two equations in Equation (10)), the scaled home CIA constraint, Equation (12), and the scaled home BC, Equation (13), taking \( x, \bar{x}, p_t(i), \bar{p}_t(\bar{i}), w_t, d_t(i), R_t, k_t, \) and \( e_t \) as given. Appendix 1 shows the first-order conditions of this maximization problem and further obtains

\[
c_t^h = \alpha(\alpha + \delta)^{-1}H_t/p_t = \alpha(\alpha + \delta)^{-1}x_t/p_t
\]

(15)

\[
c_t^b = \delta(\alpha + \delta)^{-1}H_t/(p_tz_t) = \delta(\alpha + \delta)^{-1}x_t/(p_tz_t)
\]

(16)

(17) \[ c_t^b(i) = \alpha(\alpha + \delta)^{-1}(H_t/p_t)(p_t(i)/p_t)^{-\delta} = \alpha(\alpha + \delta)^{-1}(x_t/p_t)(p_t(i)/p_t)^{-\delta} \]

(18) \[ c_t^h(i) = \alpha(\alpha + \delta)^{-1}[H_t/(p_tz_t)](\bar{p}_t(\bar{i})/\bar{p}_t)^{-\delta} = \delta(\alpha + \delta)^{-1}[x_t/(p_tz_t)](\bar{p}_t(\bar{i})/\bar{p}_t)^{-\delta}, \]

where \( H_t = m_t + (x_t - 1) + b_t - x_t b_{t+1}/R_t \) is the home individual’s money holdings available for purchasing both home and foreign goods in period \( t \). \( p_t = [\int_0^1 p_t(i)^{1-\delta}d\bar{i}]^{1/(1-\delta)} \) and \( \bar{p}_t = [\int_0^1 \bar{p}_t(\bar{i})^{1-\delta}d\bar{i}]^{1/(1-\delta)} \) are, respectively, the measure of (scaled) “average price” in home and foreign country in period \( t \) and \( z_t = k_t e_t p_t/p_t \) is the real exchange rate in period \( t \). The second equality in each of the above-mentioned four equations is obtained by applying the money and the bond market equilibrium conditions. When the money market is in equilibrium in both countries, then \( m_t = \bar{m}_t = 1 \). Moreover, since all individuals are identical, both home and foreign bonds must be in zero net supply in equilibrium, that is, for all \( t, b_t = \bar{b}_t = 0 \), and hence \( H_t = x_t \).

In Appendix 1, we also derive the following relationships:

\[
w_t = x_t H_{t+1}/[\beta(\alpha + \delta)K(e_{t+1}^h)^2(e_{t+1}^h)^\delta] = x_t x_{t+1}/[\beta(\alpha + \delta)K(e_{t+1}^h)^2(e_{t+1}^h)^\delta]
\]

(19)

\[
R_t = x_t H_{t+1}(c_t^h)^2(c_t^h)^\delta/|BH_t(e_{t+1}^h)^2(e_{t+1}^h)^\delta| = x_{t+1}(c_t^h)^2(c_t^h)^\delta/|\beta(e_{t+1}^h)^2(e_{t+1}^h)^\delta|.
\]

(20)

Similarly, for the representative foreign individual, the symmetric optimization problem can be solved following analogous reasoning to yield:

5. Note that if \( p_t(i) = \bar{p}_t \) for all \( i \), then \( p_t = \bar{p}_t \). The same property is shared by \( \bar{p}_t \).
(21) $\tilde{c}_{i}^t = \alpha(\alpha + \delta)^{-1}\tilde{H}_t/\tilde{p}_t = \alpha(\alpha + \delta)^{-1}\tilde{x}_t/\tilde{p}_t$

(22) $c_{i}^t = \delta(\alpha + \delta)^{-1}\tilde{H}_t \tilde{z}_i/\tilde{p}_t = \delta(\alpha + \delta)^{-1}\tilde{x}_t \tilde{z}_i/\tilde{p}_t$

(23) $\tilde{c}_i = \alpha(\alpha + \delta)^{-1}(\tilde{H}_t/\tilde{p}_t)(\tilde{p}(i)/\tilde{p}_t)^{-\delta}
= \alpha(\alpha + \delta)^{-1}(\tilde{x}_t/\tilde{p}_t)(\tilde{p}(i)/\tilde{p}_t)^{-\delta}$

(24) $c_{i}^t(i) = \delta(\alpha + \delta)^{-1}(\tilde{H}_t/\tilde{p}_t)(p_{i}(i)/p_i)^{-\delta}
= \delta(\alpha + \delta)^{-1}(\tilde{x}_t/\tilde{p}_t)(p_i(i)/p_i)^{-\delta}$

(25) $\tilde{w}_t = \tilde{x}_t \tilde{H}_t+1/[\beta(\alpha + \delta)K(\tilde{c}_{i}^t+1)^{\alpha}(\tilde{c}_{i}^t+1)^{\delta}]
= \tilde{x}_t \tilde{x}_t+1/[\beta(\alpha + \delta)K(\tilde{c}_{i}^t+1)^{\alpha}(\tilde{c}_{i}^t+1)^{\delta}]
\tilde{R}_t = \tilde{x}_t \tilde{H}_t+1(\tilde{c}_{i}^t)^{\alpha}(\tilde{c}_{i}^t)^{\delta}/[\beta(\tilde{H}_t+1)^{\alpha}(\tilde{c}_{i}^t+1)^{\delta}]
= \tilde{x}_t+1(\tilde{c}_{i}^t)^{\alpha}(\tilde{c}_{i}^t)^{\delta}/[\beta(\tilde{H}_t+1)^{\alpha}(\tilde{c}_{i}^t+1)^{\delta}],
\]

where $\tilde{H}_t = \tilde{m}_t + (\tilde{x}_t - 1) + \tilde{b}_t - \tilde{x}_t \tilde{b}_t+1/\tilde{R}_t$ is the money holding available for consumption to the foreign individual.

We now turn to the firms’ profit (dividend) maximization problems. Home firm $i$’s problem is to choose $p_{i}(i)$ to maximize Equation (14), subject to

(27) $p_{i}(i) = p_{i} = 0(0 - 1)^{-1}w_t$, $i \in [0, 1]$, $t = 0, 1, 2, \ldots$

A similar result holds for the foreign firms:

(29) $\tilde{p}_{i}(i) = \tilde{p}_{i} = 0(0 - 1)^{-1}\tilde{w}_t$, $i \in [0, 1]$, $t = 0, 1, 2, \ldots$

Equations (28) and (29) show that there is a marked up of price over marginal cost for firms in each country. This is due to the assumption of monopolistically competitive firms. As we know, if $\theta$ goes to infinity, then all goods tend to be perfect substitutes, and perfect competition implies $p_t = \bar{w}_t$ and $\tilde{p}_t = \tilde{w}_t$.

Finally, to fully solve for the welfare variables—$c_{i}^h$, $c_{i}^f$, and $n_{t}$ for the home individual and $\tilde{c}_{i}^h$, $\tilde{c}_{i}^f$, and $\tilde{n}_{t}$ for the foreign individual—in terms of policy variables (i.e., $x_t$ and $\tilde{x}_t$) alone, we need the following equilibrium conditions for the labor and exchange markets. The labor market clearing conditions are simply

(30) $n_{t} = c_{i}^h + c_{i}^f$, $t = 0, 1, 2, \ldots$

for the home country and

(31) $\tilde{n}_{t} = \tilde{c}_{i}^h + \tilde{c}_{i}^f$, $t = 0, 1, 2, \ldots$

for the foreign country. In Appendix 2, we show that the currency exchange market clearing condition is

(32) $x_t/p_t = z_t\tilde{x}_t/\tilde{p}_t$, $t = 0, 1, 2, \ldots$

Now we are ready to express the variables in the utility functions, Equations (8) and (9), in terms of policy variables. First, from Equations (15), (16), (21), (22), and (32), we have $c_{i}^h = (\delta/\alpha)c_{i}^h$ and $\tilde{c}_{i}^f = (\delta/\alpha)\tilde{c}_{i}^f$. Then, from Equations (30) and (31), we have $n_{t} = [(\alpha + \delta)/\alpha]c_{i}^h$ and $\tilde{n}_{t} = [(\alpha + \delta)/\alpha]\tilde{c}_{i}^f$. Thus, the issue has boiled down to expressing $c_{i}^h$ and $\tilde{c}_{i}^f$ in terms of $x_t$ and $\tilde{x}_t$. Substituting Equation (19) into Equation (28) and then the resulting expression of $p_{i}$ into Equation (15), we have

(33) $c_{i}^h = (\delta/\alpha)^{\delta}(\theta - 1)\alpha K^{-1}x_t^{-1}(c_{i}^h)^{\alpha}(\tilde{c}_{i}^f)^{\delta}$

Similarly, substituting Equation (25) into Equation (29) and then the resulting expression of $\tilde{p}_{i}$ into Equation (21), we have

(34) $\tilde{c}_{i}^f = (\delta/\alpha)^{\delta}(\theta - 1)\alpha K^{-1}\tilde{x}_t^{-1}(\tilde{c}_{i}^h)^{\alpha}(\tilde{c}_{i}^h)^{\delta}$

Denoting $Q = (\delta/\alpha)^{\delta}(\theta - 1)\alpha K/\theta$ and using log terms, we can rewrite Equations (33) and (34), respectively, as

(35) $\ln(c_{i}^h) = \ln Q + \alpha \ln(c_{i}^h) + \delta \ln(\tilde{c}_{i}^f) - \ln(x_t)$
and
\[ \ln(\tilde{c}_t) = \ln Q + 2\ln(c_{t+1}^f) + \delta\ln(c_{t+1}^h) - \ln(\tilde{x}_{t+1}). \]
\[ (36) \]

Solving this first-order difference equation system yields (see Appendix 3 for a derivation)
\[ c_t^h = Q^{1/(\alpha+\delta)}/ \left[ x_{t+1} + \sum_{j=1}^{\infty} x_{t+1+j}^{[(\alpha+\delta)^2+(\alpha-\delta)^2)/2} \sum_{j=1}^{\infty} x_{t+1+j}^{[(\alpha+\delta)^2-(\alpha-\delta)^2)/2} \right], \]
\[ (37) \]

\[ \tilde{c}_t^f = Q^{1/(\alpha+\delta)}/ \left[ \tilde{x}_{t+1} + \sum_{j=1}^{\infty} \tilde{x}_{t+1+j}^{[(\alpha+\delta)^2+(\alpha-\delta)^2)/2} \sum_{j=1}^{\infty} \tilde{x}_{t+1+j}^{[(\alpha+\delta)^2-(\alpha-\delta)^2)/2} \right]. \]
\[ (38) \]

To summarize, the home and the foreign governments' problems are, respectively, to choose \( x \) and \( \tilde{x} \) to maximize
\[ \sum_{t=0}^{\infty} |K(\alpha)|^{\delta} c_t^h \tilde{c}_t^f - (\alpha+\delta)x^{-1} c_t^h, \]
\[ (39) \]
for the home government and
\[ \sum_{t=0}^{\infty} |K(\alpha)|^{\delta} \tilde{c}_t^f \tilde{c}_t^f - (\alpha+\delta)x^{-1} \tilde{c}_t^f, \]
\[ (40) \]
for the foreign government, with \( c_t^h \) and \( \tilde{c}_t^f \) given by Equations (37) and (38), respectively.

This is a game between the home and the foreign governments. A general characterization of Nash equilibria of this game is quite complex, so we restrict our analysis to those Nash equilibria that have a constant money growth rate in each country. In Appendix 3, we prove the following proposition.

**PROPOSITION 1.** In situations in which the governments can commit to their private sectors, the strategy combination \( x_t = \tilde{x}_t = \beta \) is a Nash equilibrium if \((\alpha + \delta)(1 - \alpha)(\theta - 1)/[(\alpha(1 - \alpha) + \delta^2)0] < 1\) and the strategy combination \( x_t = \tilde{x}_t = (\alpha + \delta)(1 - \alpha)(\theta - 1)/[(\alpha(1 - \alpha) + \delta^2)0] > 1\).

The cooperative equilibria (Pareto optimal) of this game are not unique. In this article, we focus on the symmetric cooperative equilibrium in which \( x_t = \tilde{x}_t \) (and \( c_t^h = \tilde{c}_t^f \)) for all \( t = 0, 1, 2, \ldots \). In Appendix 3, we also prove that the cooperative equilibrium is \( x_t = \tilde{x}_t = \beta \), for \( t = 0, 1, 2, \ldots \). Note that this implies a steady state for all the variables in the expression for \( R_t \), and the expression for \( \tilde{R}_t \), Equation (26). So \( R_t = \tilde{R}_t = 1 \). This means that the cooperative strategy for both governments under commitment is to follow the Friedman rule, that is, to contract the money supply so that the net nominal interest rate is 0. The above-mentioned results, which are proven in Appendix 3, are summarized by the following proposition.

**PROPOSITION 2.** In the situation in which the governments can commit to their private sectors, the symmetric cooperative equilibrium of the monetary game is characterized by both governments following the Friedman rule.

From Propositions 1 and 2, we know that when \((\alpha + \delta)(1 - \alpha)(\theta - 1)/[(\alpha(1 - \alpha) + \delta^2)0] \leq 1\), the Nash equilibrium and the cooperative equilibrium coincide. So in this case, a government that is credible to its private sector does not have to worry about the actions of the other credible governments. The cooperative strategy is self-enforced. This result stands in sharp contrast with the existing literature on international monetary policy games. Most studies assume some sort of aggregate behavioral model and some kind of externality from one country’s monetary policy on another country’s economy (e.g., Canzoneri and Henderson 1991; Henderson and Zhu 1990; Kimbrough 1993). The focus of that work has been the search of rules that would implicitly coordinate different countries’ policies. However, there is an inherent incentive to cheat on any cooperative agreement or rule. On the other hand, here we find, using a natural multieconomy extension of a monetary policy game model with solid microfoundations, that

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6. Note that these Nash equilibria are the unique "constant money growth" Nash equilibria under respective conditions on parameters. There may well be other Nash equilibria that are not featured by a constant money growth rate for each government.
although a government’s monetary policy generates externalities on another economy through its impact on the exchange rate, there are situations in which the benefits of exploiting that externality are exceeded by the costs. Hence, as far as honest governments are concerned, there is no need for any rule to implement cooperation. When \((\alpha + \delta)(1 - \alpha)(0 - 1)/[(\alpha(1 - \alpha) + \delta^2)0] > 1\), however, the Nash outcome between two honest governments is less desirable than the cooperative outcome. So mechanisms that can implement cooperation do have a role to play.

The condition needed for the Friedman rule to be part of a Nash equilibrium, that is, 

\[
(\alpha + \delta)(1 - \alpha)(0 - 1)/[(\alpha(1 - \alpha) + \delta^2)0] \leq 1,
\]

depends on \(\alpha, \delta, \text{ and } 0\), about which we can provide the following explanation. First, consider the extreme case \(\delta = 0\). This is the case where individuals only consume domestic goods and the international linkage in the model is moot. The condition becomes \((0 - 1)/0 \leq 1\), which always holds. In other words, the central banks of two unlinked economies will follow the Friedman rule. This is not surprising because there is no externality to be exploited by either government in this case. Then, as \(\delta\) becomes larger while holding \(\alpha + \delta\) constant, the condition is less likely to hold because \((\alpha + \delta)(1 - \alpha)/(\alpha(1 - \alpha) + \delta^2)\) increases in \(\delta\) while holding \(\alpha + \delta\) constant. Again, this is not surprising because the stronger the international linkage, the more externality to be exploited by either government.

The overall tone of previous studies on international monetary policy game is that international linkages from trade impose a larger challenge on monetary authorities than when they are isolated and therefore require cooperation between governments. On the other hand, we have emphasized in this section that there exist situations such that honest governments, governments that can commit to their own private sectors, do not need to cooperate with each other. In the next section, we further show that in the more realistic case in which governments cannot commit to their respective private sectors, international linkages can actually improve the opportunity for the optimal monetary policy under commitment to be sustainable. Moreover, cooperation between monetary authorities may well be counterproductive.

IV. SUSTAINABLE OUTCOMES WHEN GOVERNMENTS LACK A COMMITMENT TECHNOLOGY

The analysis in the last section shows that when governments can commit to their private sectors, an optimal (cooperative) policy under commitment is for both to follow the Friedman rule, that is, to pursue a monetary policy that makes the net nominal interest rate on domestic bonds be 0. This optimal policy under commitment, however, is not the first-best policy since production in each firm of each country is conducted at a point where price exceeds marginal cost due to the monopolistically competitive market structure. So when a government lacks a commitment technology, the government has an incentive to set money growth rate unexpectedly high, after firms have fixed their nominal product prices at the beginning of each period, so that the real product price equals the marginal cost. In equilibrium, however, firms foresee this incentive of the government and set their prices accordingly. As a result, the government’s attempts to remedy the inefficiency of underproduction by generating monetary surprises lead only to higher inflation. This is, of course, a version of the time consistency problem of Kydland and Prescott (1977) and Barro and Gordon’s (1983). In the context of the present article, this point is formalized in Proposition 3.

Before stating Proposition 3, we need to introduce the concept of sustainable equilibrium and autarky plan. A sustainable equilibrium is a strategy combination such that by following the strategy combination (i) each firm in each country solves the firm’s problem, given that other agents follow the strategy combination; (ii) the representative individual in each country solves the respective individual’s problem, given that other agents follow the strategy combination; (iii) the market equilibrium requirements are satisfied; and (iv) each government solves the respective government’s problem, given that other agents follow the strategy combination. The autarky plan is

7. Increasing \(\delta\) while holding \(\alpha + \delta\) constant increases the relative weight of nondomestic goods in the utility function without changing the degree of concavity (the risk attitude) of the utility function itself.

8. See the working paper version of this article for a more formal definition of sustainable equilibrium.
a special strategy combination in which both governments always choose the maximum money growth rate \( x_t = \bar{x}_t = \bar{x} \). The subsequent proposition identifies the condition under which this autarky plan is part of a sustainable equilibrium as defined above.

**PROPOSITION 3.** If \( \varepsilon \theta \bar{x} / [(\alpha + \delta) \beta (0 - 1)] > 1 \), then the autarky plan is a sustainable equilibrium.

The proof is given in Appendix 4. Note that the condition \( \varepsilon \theta \bar{x} / [(\alpha + \delta) \beta (0 - 1)] > 1 \) can always be satisfied since \( \bar{x} \) can be arbitrarily large. In the autarky equilibrium, \( x_t = \bar{x}_t = \bar{x} \) for all \( t \). This is the counterpart of discretionary outcomes by Barro and Gordon (1983).

Therefore, when governments lack a commitment technology, the question is not whether the first best can be achieved but rather whether the second best, the optimal allocation under commitment—the allocation that is realized when each government follows the Friedman rule—can be sustained. Ireland (1997) investigated this question within a closed-economy model characterized by utility-maximizing households, monopolistically competitive firms, and sticky nominal product prices. Specifically, following the technique developed by Abreu (1988) and Chari and Kehoe (1990), Ireland established a full characterization of “sustainable outcomes,” outcomes that can be supported by “sustainable equilibria,” when the government lacks a formal commitment technology. He went on to identify the conditions under which the optimal policy under commitment—the Friedman rule—can be part of a sustainable outcome.

The model we consider here is a two-country version of Ireland’s model. For simplicity, we will not use the Abreu-Chari-Kehoe technique to fully characterize the sustainable outcomes first. Instead, we directly establish the condition for the Friedman rule to be sustainable and compare it with the similar condition by Ireland for the closed economy.\(^9\) For the Friedman rule—the cooperative equilibrium under commitment \( (x_t = \bar{x}_t = \bar{x}) \)—to be sustainable, it must be the case that for each government, the benefit from deviation is exceeded by the cost of it. Because the autarky plan is itself a sustainable equilibrium, it can be credibly used to punish any deviation. So the cost of deviating from the Friedman rule is that from the next period on, both the two governments and the private agents revert to the autarky plan in which \( x_t = \bar{x}_t = \bar{x} \). On the other hand, the benefit from a government deviating from the Friedman rule is that by setting the gross money growth rate larger than \( \beta \), after the firms have set prices according to both governments setting money growth rate to \( \beta \), the inefficiency in the product market can be partially fixed and the instant one-period utility of the representative domestic individual can be improved.

Because of symmetry, we need to consider only the home government’s incentive to deviate. Specifically, we compare the home individual’s lifetime utility obtained when the home government follows the Friedman rule with that obtained when the home government deviates. The constant one-period utility the home individual gets under the Friedman rule \( (x_t = \bar{x}_t = \bar{x}) \) is, letting \( \bar{x} = \bar{x} = \bar{x} \) in (A3.6),

\[
K(\delta/\alpha)^{(Q/\beta)\frac{1}{1-(\alpha+\delta)}-1} - (\alpha + \delta)(Q/\beta)\frac{1}{1-(\alpha+\delta)}
\]

\[
= K^{1-(\alpha+\delta)/(\alpha+\delta)} \frac{(Q/\beta)^{1-(\alpha+\delta)}}{1-(\alpha+\delta)} \left\{ \left[ (0 - 1) / \bar{x} \right] \left[ (0 - 1) / \bar{x} \right] \right\}.
\]

(41)

Similarly, the one-period utility the home individual gets under the autarky plan is

\[
K(\delta/\alpha)^{(Q/\bar{x})\frac{1}{1-(\alpha+\delta)}-1} - (\alpha + \delta)(Q/\bar{x})\frac{1}{1-(\alpha+\delta)}
\]

\[
= K^{1-(\alpha+\delta)/(\alpha+\delta)} \frac{(\bar{x}/\beta)^{(\alpha+\delta)/(\alpha+\delta)}}{1-(\alpha+\delta)} \left\[ \left[ (\bar{x}/\beta)^{(0 - 1) / \bar{x}} \right] \left[ (0 - 1) / \bar{x} \right] \right\}.
\]

(42)

If deviating, the home government would choose \( x_t \) to maximize the home individual’s instant one-period utility at \( t \),

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\(^9\) The Abreu-Chari-Kehoe technique is used to fully characterize the sustainable outcomes, and from there, to derive the condition for the Friedman rule to be sustainable, in the working paper version of this article, which is available upon request.
\[ K(\delta/\alpha)^\delta (c_t^h)^{\alpha} (c_t^f)^{\delta} - (\alpha + \delta)\alpha^{-1}c_t^h \]
\[ = K(\delta/\alpha)^\delta [(\alpha + \delta)^{-1}\alpha x_t/p_t]\]^{\alpha} [(\alpha + \delta)^{-1}\alpha x_t/p_t]^{\delta} - x_t/p_t, \quad (43) \]
given that \( x_t = \beta \), and firms in both countries set prices, assuming that both governments follow the Friedman rule. To get \( p_t \) and \( \tilde{p}_t \) in Equation (43), note that (from (A3.5)) under \( x_t = \tilde{x}_t = \beta \), \( c_t^h = c_t^f = (Q/\beta)^{(\alpha+\delta)/\alpha} \). On the other hand, \( c_t^h = \beta(\alpha + \delta)^{-1}/p_t \) and \( c_t^f = \beta(\alpha + \delta)^{-1}/\tilde{p}_t \) from Equations (15) and (21). So
\[ p_t = \tilde{p}_t = \beta(\alpha + \delta)^{-1}(Q/\beta)^{1/(\alpha+\delta)}. \quad (44) \]

Substituting \( p_t \) and \( \tilde{p}_t \) given by Equation (44) and \( \tilde{x}_t = \beta \) into Equation (43), the latter becomes
\[ K(\delta/\alpha)^\delta \beta^{-2}(Q/\beta)^{1/(\alpha+\delta)} x_t^\alpha - (\alpha + \delta)(\alpha \beta)^{-1}(Q/\beta)^{1/(\alpha+\delta)} x_t^\alpha. \quad (45) \]

Maximizing Equation (45) with respect to \( x_t \) yields the first-order condition
\[ x_t^{1-\alpha} = \beta^{1-\alpha}\alpha(\alpha + \delta)(\alpha + \delta - 1)]. \quad (46) \]

To see whether it is worthwhile for the home government to deviate, we differentiate between two cases depending on the value of \( \alpha \) relative to \( \alpha/\delta \).

**CASE (I).** \( \alpha \geq 1 + \alpha/\delta \).

In this case, Equation (46) would indicate that \( x_t \leq \beta \). Because the lower bound of \( x_t \) is \( \beta \), we have a corner solution \( x_t^* = \beta \). This means, when firms produce close enough substitutes (\( \alpha \geq 1 + \alpha/\delta \)), given \( \alpha/\delta \), a government cannot even generate any instant benefits by deviating from the Friedman rule if the other government is still following the rule. So the Friedman rule is sustainable.

Why is the elasticity of substitution so important in determining the sustainability of the optimal policy? The larger this elasticity, the less the monopoly distortion introduced by imperfect goods market competition. In other words, the two governments’ incentives to deviate from the Friedman rule is lower the higher is the elasticity of substitution and the lower the degree of monopoly distortion. Also note that as the weight of foreign goods in the home individual’s utility function becomes smaller, the lower bound on \( \theta \) would increase. This result is also intuitive because a larger portion of goods market inefficiency will be eliminated in the short run by the home government’s deviation if foreign goods weigh less in the home individual’s utility function. For a deviation not to generate any instant benefits, the whole size of the goods market inefficiency must be smaller (\( \theta \) is larger).

It has been noted that the time consistency problem for monetary policy vanishes when the nonmonetary distortion is removed (Blackburn and Christensen 1989; Rogoff 1989). Here, we show a case that even with a nonmonetary distortion, the time consistency problem will not come up because one country’s monetary policy alone can do nothing to fix the distortion. This point is related to the work by Cukierman and Drazen (1988), who argued that if tax revenue from distortionary income taxes is used to provide public goods, it may not be true that unanticipated monetary policies would necessarily have short-run benefits. Our case, however, comes out of a basic model in which unanticipated monetary policy has been proved to have temporary benefits due to the improved production efficiency (Ireland 1997). By extending Ireland’s model to a multieconomy world, we show that there are situations (\( \theta \geq 1 + \alpha/\delta \)) where the governments of interacting economies have to coordinate to make monetary surprises able to generate short-run production efficiency. However, there is no reason to believe that two different governments can be easily engaged in such coordination. To the contrary, coordination failure between the governments provides a mechanism to implement the Friedman rule.

Here, we present situations where lack of a coordination mechanism ensures sustainability of the Friedman rule, while existence of such mechanisms may render the Friedman rule not sustainable (depending on short-run benefits from deviation and long-run cost from loss of reputation), resulting in inflation biases. These situations where the governments of interacting economies have to coordinate to make monetary surprises able to generate short-run production efficiency facilitate a variation of the paradoxical result of counterproductive cooperation between two governments by Rogoff (1985) and Kehoe (1989). Their basic insight is that commitment
between two governments may be counterproductive when commitment with respect to the private sector is not available. However, the counterproductive effect of cooperation between governments in our model is of a different form than in Rogoff (1985) and Kehoe (1989). Here, the optimal policy becomes less sustainable (in a repeated game) due to the cooperation between governments, while in Rogoff (1985) and Kehoe (1989), it is the time-consistent suboptimal policy (in a one-shot game) that becomes worse due to cooperation. In a recent article by Jensen (1997), it is shown that while a costly delegation of monetary policy can improve time-consistent suboptimal outcomes in one-shot games—as predicted by the conventional wisdom on the role of delegation in alleviating the consequences of the time inconsistency problem—it makes the sustainability of the optimal monetary policy worse in repeated play settings.

Is $0 \leq 1 + \alpha / \delta$ plausible? This condition implies a markup of $0 / (\theta - 1) \leq 1 + \alpha / \delta$. Based on the review of the markup literature by Rotemberg and Woodford (1992), the preferred value for markup is 1.2, and it is not likely that it would exceed 2. These two markup values correspond to $\delta / \alpha = 0.2$ and $\delta / \alpha = 1$, respectively. So our Case (I) seems to be identified by the literature as a plausible case for sufficiently open economies.

**CASE (II).** $0 < 1 + \alpha / \delta$.

In this case, we have an interior solution $x^* = (\alpha 0 / (\alpha + \delta) (\theta - 1))^{1/2} > \beta$. Hence, the instant one-period utility generated by deviating from the Friedman rule is, letting in Equation (45) $x_t = x^*_t$,

$$K^{1-\gamma}(\delta / \alpha)^{\delta / (\alpha + \delta)} \alpha^{1-\gamma}(\alpha + \delta)^{\alpha / (\alpha + \delta)}(\alpha / (\alpha + \delta))^{\gamma / \gamma}$$

$$\{ \alpha^{-1} [ (\theta - 1) / \theta ]^{1-\gamma} [ (\theta - 1) / \theta ]^{1-\gamma} \}$$

$$- \left[ (\theta - 1) / \theta \right]^{1-\gamma} [ (\theta - 1) / \theta ]^{1-\gamma} \}.$$  

Comparing this result with that in Case (I), we can say that when the nonmonetary distortion is moderate, a government cannot enhance the production efficiency even for a single period using monetary surprises without coordinating with the other government. On the other hand, if the nonmonetary distortion is severe, a government can increase short-run efficiency by acting alone. However, even in this case, whether the governments will deviate depends on the trade-off between the short-run benefits from the temporary increase in production efficiency and the long-run costs that would incur due to a loss of reputation. Specifically, the necessary and sufficient conditions for not deviating, or put the other way, the necessary and sufficient condition for the Friedman rule to be sustainable is, letting $\bar{x}$ be arbitrarily large,

$$(1 - \beta) \left[ (\alpha / (\alpha + \delta))^{\gamma / \gamma} [ (\theta - 1) / \theta ]^{1-\gamma} \right]$$

$$+ \frac{\alpha + \delta}{\alpha + \delta} \left[ (\theta - 1) / \theta \right]^{1-\gamma} [ (\theta - 1) / \theta ]^{1-\gamma}.$$  

(48)

Compare this condition with Ireland’s condition for the closed-economy model,

$$\left(1 - \beta\right) \left(1 / \alpha - 1 \right) \leq \alpha^{-1} [ (\theta - 1) / \theta ]^{2/2}$$

$$- \left[ (\theta - 1) / \theta \right]^{2/2}.$$  

(49)

Obviously, as $\delta$ approaches 0, Condition (48) converges to Condition (49). This is the case because our two-country open-economy model nests Ireland’s (1997) closed-economy model as a special case in the limit as $\delta$ approaches 0. For $\delta > 0$, we show in Appendix 5 that Condition (48) is weaker than Condition (49) for comparable parameters; therefore, the Friedman rule is more likely to be sustainable in our open-economy model than in Ireland’s closed-economy model. Summarizing the results in Case (I) and Case (II) yields the following proposition.

**PROPOSITION 4.** For given parameter values, the Friedman rule is more likely to be sustainable in an open-economy environment than in the closed-economy environment. Moreover, in the open-economy environment, and when the firms in the economy produce reasonably close substitutes ($0 \geq 1 + \alpha / \delta$), a government’s deviation from the Friedman rule without coordinating
with the other government would not generate even the short-run (or instant) benefits.

The source of the instantaneous gain one government gets by deviating from the Friedman rule is the production inefficiency associated with a monopolistically competitive market structure. The larger the instantaneous gain from deviation, the less likely the optimal policy is to be sustainable. While a government in a closed economy can fully exploit these instantaneous benefits from deviating, a unilateral decision by one government in an open economy cannot fully achieve such benefits. As a result, the optimal monetary policy becomes more likely to be sustainable in an open-economy setting.

Proposition 4 highlights the importance of the elasticity of substitution between goods of the same origin in determining the sustainability of the Friedman rule. An interesting question is how the elasticity of substitution between the group of domestic goods and that of foreign goods would affect the sustainability of the Friedman rule. In principle, this new dimension can be added to our model by simply replacing the Cobb-Douglas specification of the utility function (see Equations (8) and (9)), which implies a unitary elasticity of substitution between these two groupings of goods, with a CES specification. It is our conjecture that the larger the elasticity of substitution between these two baskets of goods, the easier for the Friedman rule to be sustainable. Nevertheless, formally demonstrating that assertion seems to be a daunting task and one which we have not been able to pursue here.

V. CONCLUSIONS

There have been many studies on monetary policy games. A new trend in this research area is to revisit the issue of monetary policy effectiveness and time consistency problem with models that are based on micro-level behavior specifications of private agents and the assumption that the government’s objective is to maximize social welfare defined on individual utilities (Cubitt 1993; Ireland 1996, 1997). This article is another attempt in this direction, in which we add a second economy to Ireland’s (1997) closed-economy model with utility-maximizing households, monopolistically competitive firms, and sticky goods prices. We study the game between two governments and between the governments and their private sectors at the same time. Our results are very different from those obtained previously using ad hoc specifications for aggregate relations and governments’ objective functions. First, ignoring governments’ commitment problems, cooperation between governments may be or may not be a problem in our model—the cooperative equilibrium may be self-enforcing, depending on the values of the parameters of the economy. Second, explicitly taking governments’ commitment problems into account, we find that interactions between economies would make each government’s monetary promises more credible; hence, the Friedman rule would be more likely to be sustainable.

The main message of this article is quite positive: free trade can help alleviate the commitment problem. In essence, the government of an isolated economy, even a benevolent one, has too much power in terms of controlling real prices to remedy, in the short run, the nonmonetary inefficiency caused by a monopolistically competitive market structure. This power turns out to be not only powerless in fixing the nonmonetary distortion but also the source of a monetary distortion. In contrast, when two economies are connected through trade, each government can control the real price (in the short run) of only a fraction of all the goods its citizens consume and therefore can offset the monopolistic competition distortion in only a fraction of the market. Using this reasoning, any other changes in economic environments that reduce a government’s power in controlling real prices would also help alleviate the commitment problem and hence make the optimal monetary policy sustainable.

11. In our model, each firm must set a single nominal price, denominated in units of its own country’s currency, for its output sold in either country. This assumption, coupled with the flexibility of the nominal exchange rate, implies that home individuals face fixed nominal prices for domestic goods but fully flexible nominal prices for imported goods. An alternative pricing assumption would allow each firm to preset two prices: one for its output in the domestic market denominated in the domestic currency and the other for its output in the foreign market denominated in the foreign currency. Then, home individuals would face fixed nominal prices for both domestic and imported goods. However, the added price variables would make the notation significantly more complicated. Nevertheless, even under that alternative pricing assumption, each government could still only control the real prices for a fraction of the goods produced and consumed in the world market—the goods consumed by domestic individuals under the alternative pricing assumption as compared to the goods produced by domestic firms under the current pricing assumption.
In this article, the international linkage is made through trade alone. In particular, we have ruled out individuals holding foreign bonds. A consequence of this assumption is that the need to keep the trade balance in equilibrium every period forces the exchange rate to adjust on impact, thus isolating the imported goods from the domestic surprise inflation. In reality, international lending and borrowing are other important dimensions of country openness. If foreigners hold a substantial fraction of domestic currency (be it bonds or currency), deviations from the Friedman rule could have a substantial payoff.\textsuperscript{12} While we have made the assumption of no holding of foreign bonds for analytical tractability, relaxing this assumption and allowing for trade deficits or surpluses may be a potential topic for future research.

It would also be interesting to see if our results are invariant with various other specifications of monetary policy games that have solid microfoundations. For example, Cubitt (1993) used a different model to study the time consistency problem in which menu costs, instead of cash in advance constraints, contribute to the cost of inflation, and both product market imperfection and labor market monopoly cause the nonmonetary distortion. So the exact implications of our model and results for resolving real-world problems of time consistency in monetary policy remains a topic that warrants further study.

\textbf{APPENDIX 1}

Assume that both the home CIA constraint, Equation (12), and the home BC, Equation (13), hold with equality. The Lagrangian of the home individual’s problem is

\[
L = \sum_{t=0}^{\infty} \beta^t [K(c_t^h)\delta - n_t] + \sum_{j=0}^{\infty} \beta^j \left[ m_t + (x_t - 1) + b_t - x_t h_{t+1}/R_t - \left( \int_0^1 p_t(i) c_{t+1}^h(i) \, di + k_t c_t(i) \int_0^1 \tilde{p}_t(i) c_t^h(i) \, di \right) \right] + \sum_{j=0}^{\infty} \beta^j \left[ m_t + (x_t - 1) + b_t - x_t h_{t+1}/R_t + n_t w_t \right] + \int_0^1 d_t(i) \, di - \left( \int_0^1 p_t(i) c_t^h(i) \, di + k_t c_t(i) \int_0^1 \tilde{p}_t(i) c_t^h(i) \, di \right) + x_t m_{t+1}. \]

\textsuperscript{12} This is the case that most governments seem to have in mind when considering the advantages of devaluations as a way to reduce the real value of the domestic debt held by foreigners.

Hence, the first-order conditions of the home individual’s problem are

\begin{align*}
\dot{c}_t^h(i) &= \partial K(c_t^h)\delta - n_t = (\lambda_t + \mu_t) p_t(i) \\
\dot{c}_t^h(i) &= \delta K(c_t^h)(c_t^h(i))^{1/\delta} - 1 = k_t c_t(i) (\lambda_t + \mu_t) \tilde{p}_t(i) \tag{A1.1} \\
\dot{n}_t &= \lambda_t w_t = 1 \tag{A1.3} \end{align*}

\begin{align*}
m_{t+1} &= \beta(\mu_{t+1} + \lambda_{t+1}) = \lambda_{t+1} x_t \tag{A1.4} \\
b_{t+1} &= \beta(\mu_{t+1} + \lambda_{t+1}) = (\mu_t + \lambda_t) x_t/R_t. \tag{A1.5} \end{align*}

By taking the integral on both sides of (A1.1) to the power of \((1 - \delta)\) with respect to \(i\) from 0 to 1 and applying the definitions of \(c_t^h\) and \(p_t\), we have

\[
\partial K(c_t^h)\delta - n_t = (\lambda_t + \mu_t) p_t z_t, \tag{A1.6} \]

where \(z_t = k_t c_t(i) \tilde{p}_t(i).\)

Letting \(H_t = m_t + (x_t - 1) + b_t - x_t h_{t+1}/R_t\), the home CIA constraint, Equation (12), becomes

\[
H_t = \int_0^1 p_t(i) c_t^h(i) \, di + k_t c_t(i) \int_0^1 \tilde{p}_t(i) c_t^h(i) \, di. \tag{A1.8} \]

Multiplying both sides of (A1.8) by \((\lambda_t + \mu_t)\) and then substituting (A1.1) and (A1.2) into it, we have

\[
(\lambda_t + \mu_t) H_t = (\alpha + \delta) K(c_t^h)(c_t^h)\delta. \tag{A1.9} \]

From (A1.6), (A1.7), and (A1.9), we have

\[
c_t^h = (\alpha + \delta)^{-1} H_t/p_t, \tag{A1.10} \]

\[
c_t^h = \delta(\alpha + \delta)^{-1} H_t/(p_t z_t). \tag{A1.11} \]

Now substituting (A1.9)–(A1.11) back into (A1.1) and (A1.2) yields

\[
c_t^h(i) = (\alpha + \delta)^{-1} (H_t/p_t)_{i=p_t} \tag{A1.12} \]

\[
\dot{c}_t^h(i) = \delta(\alpha + \delta)^{-1} (H_t/p_t z_t)_{i=p_t}. \tag{A1.13} \]

Finally, (A1.3), (A1.4), and (A1.9) together give us

\[
w_t = x_t H_{t+1}/[\beta(\alpha + \delta) K(c_t^h)(c_t^h)\delta], \tag{A1.14} \]

and (A1.5) and (A1.9) together give us

\[
R_t = x_t H_{t+1}/[\beta(\alpha + \delta) K(c_t^h)(c_t^h)\delta]. \tag{A1.15} \]
APPENDIX 2

The demand in period t for foreign money by the representative home individual is

\[
\int_0^1 P_t(z)z^b(z)\,dz = M_t^z P_t^z z_t/(P_t^z z_t),
\]

and the supply of foreign money by the representative foreign individual is

\[
\left(1/e_t\right) \int_0^1 P_t(z)z^b(z)\,dz = M_t^z P_t^z z_t/(e_t P_t^z).
\]

Note that the foreign exchange market equilibrium condition with respect to home money is also (A2.3) due to Walras Law.

APPENDIX 3

3.1 Solving the Difference Equation System

To solve the difference equation system,

\[
\ln(c^b_t) = \ln Q + \alpha \ln(c^x_{t+1}) + \delta \ln(c^x_{t+1}) - \ln(x_{t+1})
\]

\[
\ln(c^f_t) = \ln Q + \alpha \ln(c^x_{t+1}) + \delta \ln(c^x_{t+1}) - \ln(\tilde{x}_{t+1})
\]

we first add them up, which yields

\[
\ln(c^b_t) + \ln(c^f_t) = 2\ln Q + (\alpha + \delta) \left[\ln(c^x_{t+1}) + \ln(c^x_{t+1})\right]
- \ln(x_{t+1}) - \ln(\tilde{x}_{t+1}) - \ln(\tilde{x}_{t+1})
\]

or

\[
\ln(c^b_t) + \ln(c^f_t) = 2\ln Q/\left[1 - (\alpha + \delta)\right] - \ln(x_{t+1}) - \ln(\tilde{x}_{t+1}) - \ln(\tilde{x}_{t+1})
\]

From (A3.1) and (A3.2), we have

\[
\ln(c^b_t) = \ln Q/[1 - (\alpha + \delta)] - \ln(x_{t+1}) - \ln(\tilde{x}_{t+1}) - \ln(\tilde{x}_{t+1})
\]

or

\[
\ln(c^b_t) = \ln Q/[1 - (\alpha + \delta)] - \ln(x_{t+1}) - \ln(\tilde{x}_{t+1}) - \ln(\tilde{x}_{t+1})
\]

3.2 Constant Money Growth Nash Equilibrium

From (A3.3) and (A3.4), when both governments follow a constant growth rate monetary policy, \(x_t = \tilde{x}\) for all t and \(\tilde{x}_t = \tilde{x}\) for all t, and \(c^b_t\) and \(c^f_t\) are also constant over time and are, respectively,

\[
c^b_t = Q^{1/(\alpha + \delta)} \tilde{x}_t^{-1}
\]

\[
c^f_t = Q^{1/(\alpha + \delta)} \tilde{x}_t^{-1}
\]

We then subtract the second one from the first one, which yields

\[
\ln(c^b_t) - \ln(c^f_t) = (\alpha + \delta) \left[\ln(c^x_{t+1}) - \ln(c^x_{t+1})\right]
- \ln(x_{t+1}) - \ln(\tilde{x}_{t+1}) - \ln(\tilde{x}_{t+1})
\]

or

\[
\ln(c^b_t) - \ln(c^f_t) = (\alpha + \delta) \left[\ln(c^x_{t+1}) - \ln(c^x_{t+1})\right]
- \ln(x_{t+1}) - \ln(\tilde{x}_{t+1}) - \ln(\tilde{x}_{t+1})
\]

\[
u^b_t = K(\delta/\alpha)^{\delta} [c^b_t]^{-(1-\alpha)} - (\alpha + \delta)^{-(1-\alpha)} c^b_t
\]

\[
u^f_t = K(\delta/\alpha)^{\delta} [c^f_t]^{-(1-\alpha)} - (\alpha + \delta)^{-(1-\alpha)} c^f_t
\]

\[
u^b_t = K(\delta/\alpha)^{\delta} [c^b_t]^{-(1-\alpha)} - (\alpha + \delta)^{-(1-\alpha)} c^b_t
\]

\[
u^f_t = K(\delta/\alpha)^{\delta} [c^f_t]^{-(1-\alpha)} - (\alpha + \delta)^{-(1-\alpha)} c^f_t
\]

\[
u^b_t = K(\delta/\alpha)^{\delta} [c^b_t]^{-(1-\alpha)} - (\alpha + \delta)^{-(1-\alpha)} c^b_t
\]
\[-(\alpha + \delta)\alpha^{-1}Q^\frac{1}{1-(\alpha+\delta)}_{1-(\alpha+\delta)[1-(\alpha+\delta)-b]} \chi[1-(\alpha+\delta)[1-(\alpha+\delta)]].\]

\[(A3.6)\]

Maximizing (A3.6) (which is equivalent to maximizing the lifetime utility) with respect to \(x\), the first-order condition is

\[(A3.7) \quad \hat{x} = (\alpha + \delta)(1-\alpha)(0-1)\beta/[(\alpha(1-\alpha) + \delta^2)0] \leq 1.\]

In this case, the optimal strategy of the home government is \(\hat{x} = \beta\). This is a corner solution. By symmetry, the optimal strategy of the foreign government is \(\hat{x}_f = \beta\). So \(\hat{x} = \hat{x}_f = \beta\) is a Nash equilibrium.

**CASE I.** \((\alpha + \delta)(1-\alpha)(0-1)/[(\alpha(1-\alpha) + \delta^2)0] > 1.\)

In this case, \(\hat{x}\) has an interior solution \(\hat{x} = (\alpha + \delta)(1-\alpha)(0-1)\beta/[(\alpha(1-\alpha) + \delta^2)0]\). Similarly, the optimal strategy of the foreign government is \(\hat{x}_f = (\alpha + \delta)(1-\alpha)(0-1)\beta/[(\alpha(1-\alpha) + \delta^2)0]\). So \(\hat{x} = \hat{x}_f = (\alpha + \delta)(1-\alpha)(0-1)\beta/[(\alpha(1-\alpha) + \delta^2)0] > 1\) is a Nash equilibrium.

This completes the proof of Proposition 1.

### 3.3 Symmetric Cooperative Equilibrium

Now we derive the symmetric cooperative equilibrium of the game between two honest governments. The problem facing the two cooperative governments is to choose \(\hat{x} = \hat{x}_f\) for all \(t\) (hence, \(c^t_f = c^t\)), to maximize the one-period utility (therefore the lifetime utility) of the representative individual in each country, which is

\[(A3.8) \quad u(c_t) = K(\delta/\beta)^{\frac{1}{1-(\alpha+\delta)}} - (\alpha + \delta)x^{-1}c_t,\]

where

\[(A3.9) \quad c_t = \frac{Q^\frac{1}{1-(\alpha+\delta)}_{1-(\alpha+\delta)}}{1-(\alpha+\delta)}.\]

It is easy to see \(u'(c_t) < 0\), and the solution to \(u'(c_t) = 0\) is

\[(A3.10) \quad c^* = [K(\delta/\beta)^{\frac{1}{1-(\alpha+\delta)}}].\]

So the symmetric cooperative equilibrium of this game between these two governments is \(x_t = \hat{x}_t = \beta\), \(t = 0, 1, 2, \ldots\).

This completes the proof of Proposition 2.

### APPENDIX 4

By the very definition of the autarky plan, to prove that it is a sustainable equilibrium, we only need to prove that it is each government’s optimal policy to follow this plan if the other government and private agents all follow the plan.

Taking \(x_t = \hat{x}_t = \beta\), the home individual would have (from (A3.5))

\[c_0^t = Q^\frac{1}{1-(\alpha+\delta)}_{1-(\alpha+\delta)} = [(\delta/\beta)^{\delta}bK(0-1)/(0\hat{x})]^{\frac{1}{1-(\alpha+\delta)}}.\]

On the other hand, \(c_0^t\) can also be obtained through Equation (15), which says \(c^t_0 = x(\alpha + \delta)^{-1}\hat{x}_f/p\), where \(p\) is the constant (with respect to \(t\)) price level in both countries set by all the firms taking \(x_t = \hat{x}_t = \beta\).

**Given \(p\) as above, and \(\hat{x}_t = \beta\) for all \(t\), we want to establish that \(x_t = \hat{x}_t = \beta\) for all \(t\) maximizes the one-period utility (hence the lifetime utility) of the home individual**

\[(A4.1) \quad \bar{u} = x(\alpha + \delta)^{-1}[(\delta/\beta)(0-1)bK/(0\beta)]^{\frac{1}{1-(\alpha+\delta)}}.\]

Given \(\bar{p}\) as above, and \(\hat{x}_t = \beta\) for all \(t\), we want to establish that \(x_t = \hat{x}_t = \beta\) for all \(t\) maximizes the one-period utility (hence the lifetime utility) of the home individual

\[(A4.2) \quad u^t_0 = (\delta/\beta)^{\frac{1}{1-(\alpha+\delta)}}(\hat{x}^t_0 - (\alpha + \delta)x^{-1}c^t_0),\]

where \(c^t_0 = x(\alpha + \delta)^{-1}\hat{x}_f/p\) and \(\hat{x}^t_0 = x(\alpha + \delta)^{-1}\hat{x}_f/p\) according to Equations (15) and (21).

It is easy to see \(\partial^2u^t_0/\partial x^2 < 0\), and

\[\partial^2u^t_0/\partial x^2 = (1/p)[x(\alpha + \delta)^{-1}(\delta/\beta)^{\delta}aK[x(\alpha + \delta)^{-1}x/p]^{\delta-1} - (\alpha + \delta)^{-1}x/p]^{\delta-1} - 1],\]

\[(A4.3) \quad \partial^2u^t_0/\partial x^2 = (1/p)[x(\alpha + \delta)^{-1}(\delta/\beta)^{\delta}aK[x(\alpha + \delta)^{-1}x/p]^{\delta-1} - 1].\]

\[(A4.4) \quad \partial^2u^t_0/\partial x^2 = (1/p)[x(\alpha + \delta)^{-1}(\delta/\beta)^{\delta}aK[x(\alpha + \delta)^{-1}x/p]^{\delta-1} - 1].\]

So as long as \(a0\beta/[(\alpha + \delta)(0-1)] > 1, x_t = \hat{x}_t = \beta\) for all \(t\) would be the home government’s optimal policy if the other government and private agents follow the autarky plan.

### APPENDIX 5

Let \(\alpha\) in Condition (49) be \(\alpha + \delta\) so that parameters in the two models are comparable. Then, Condition (49) becomes

\[(A5.1) \quad (1-\beta)[1-(\alpha + \delta)] \leq [(0-1)/\theta]^{\frac{\delta}{1-(\alpha+\delta)}}.\]

To prove that Condition (48) is weaker than Condition (A5.1), it is equivalent to show that

\[(A5.2) \quad \alpha[\alpha/(\alpha + \delta)]^{\frac{\alpha}{\alpha-\delta}}[0/(0-1)]^{\frac{1}{1-(\alpha+\delta)}} - (\alpha + \delta) < 1\]

for all \(\alpha > 0, \delta > 0, \alpha + \delta < 1, \) and

\[(A5.3) \quad \theta^0/[(\alpha + \delta)(0-1)] > 1\]

(i.e., \(0 < 1 + \alpha/\delta, \) the Case (II) requirement).

Note that the left side of (A5.2) is strictly decreasing in \(\theta/(0-1)\). Therefore, to show that (A5.2) holds for all \(\theta/(0-1)\).
that satisfy (A5.3), it is sufficient to show that it holds when $0/(0-1) = (\alpha + \delta)/\alpha$, or

\[(A5.4) \quad [\alpha/(\alpha + \delta)] - [\alpha/(\alpha + \delta)]^2 (1 - \alpha) \leq 1 - (\alpha + \delta).\]

It can be easily checked by signing the first-order derivative that the left side of (A5.4) is increasing in $\alpha$ while holding $\alpha + \delta$ fixed. Therefore, to show (A5.4) holds, it is sufficient to show that it holds as $\alpha$ increasingly approaches $\alpha + \delta$, which is $1/(\alpha + \delta) \leq 1 - (\alpha + \delta)$, a true statement.

REFERENCES


